TRUE TRANSFORMATIONS OF SPACETIME LENGTHS AND APPARENT TRANSFORMATIONS OF SPATIAL AND TEMPORAL DISTANCES. II. THE COMPARISON WITH EXPERIMENTS

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Some of the well-known experiments: the "muon" experiment, the Michelson-Morley type experiments, the Kennedy-Thorndike type experiments and the Ives-Stilwell type experiments are analyzed using the nonrelativistic theory, the "apparent transformations (AT) relativity" and the "true transformations (TT) relativity." It is shown that all the experiments (when they are complete from the "TT relativity" viewpoint) are in agreement with the "TT relativity" in which the special relativity is understood as the theory of a four-dimensional spacetime with the pseudo-Euclidean geometry. It is also explicitly shown that, in contrast to the usual opinion, the commonly used "AT relativity" does not always agree with experiments. The concept of sameness of a physical quantity is essential for the distinction between the two forms of relativity both in the theory and in experiments. The difference in this concept causes the agreement of the "TT relativity" with the experiments and the disagreement of the "AT relativity."

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Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows and only a kind of union of the two will preserve an independent reality. H. Minkowski

A quantity is therefore physically meaningful (in the sense that it is of the same nature to all observers) if it has tensorial properties under Lorentz transformations. F. Rohrlich

I. INTRODUCTION

In [1], [2] and [3] (this paper will be referred as $[I]$), (see also [4]), two forms of relativity are discussed, the "true transformations (TT) relativity" and the "apparent transformations (AT) relativity." The notions of the TT and the AT are first introduced by Rohrlich [5], and, in the same meaning, but not under that name, discussed in [6] too. The general theoretical discussion of the difference between the "TT relativity" and the "AT relativity" is given in detail in $[I]$. There (in $[I]$) we have also presented the theoretical discussion of the TT of the spacetime length for a moving rod and a moving clock, and of the AT for the same examples, i.e., the AT of the spatial distance, the Lorentz "contraction," and the AT of the temporal distance, the time "dilatation." In this paper we use theoretical results from $[I]$ and compare them with some experimental results.

II. GENERAL DISCUSSION OF THE COMPARISON

It is usually interpreted that the experiments on "length contraction" and "time dilatation" test the special relativity, but the discussion from $[I]$ shows that such an interpretation of the experiments refers to - the "AT relativity," and not to - the "TT relativity."

It has to be noted that in the experiments in the "TT relativity," in the same way as in the theory, see $[I]$, the measurements in different inertial frames of reference (IFRs) (and different coordinatizations) have to refer to the same four-dimensional (4D) tensor quantity. In the chosen IFR and the chosen coordinatization the measurement of some 4D quantity has to contain the measurements of all parts of such a quantity. However in almost all experiments that refer to the special relativity only the quantities belonging to the "AT relativity" were measured. From the "TT relativity" viewpoint such measurements are incomplete, since only some parts of a 4D quantity, not all, are measured. This fact presents a serious difficulty in the reliable comparison of the existing experiments with the "TT relativity," and, actually, we shall be able to compare in a quantitative manner only some of the existing experiments with the "TT relativity."

To examine the differences between the nonrelativistic theory, the commonly used "AT relativity," and the "TT relativity" we shall make the comparison of these theories with some experiments in the following sections.

III. THE "MUON" EXPERIMENT

First we shall examine an experiment in which different results will be predicted for different synchronizations in the conventional approach to relativity, i.e., in the "AT relativity," but of course the same results for all synchronizations will be obtained in the "TT

relativity." This is the "muon" experiment, which is already theoretically discussed from the "TT relativity" viewpoint in Sec. 3.2 in $[I]$ and from the "AT relativity" viewpoint in Sec. 4.2 in $[I]$. The "muon" experiment is quoted in almost every text-book on general physics, see, e.g., [7] and [8]. Moreover, an experiment [9] was the basis for a film often shown in introductory modern physics courses: "Time dilation: An experiment with μ mesons." Recently, in [10], a version of such an experiment is presented, and it required travelling to mountains of moderate heights of around 600 m.

In these experiments, [9] and [10], the fluxes of muons on a mountain, N_m, and at sea level, N_s, are measured, and the number of muons which decayed in flight is determined from their difference. Also the distribution of the decay times is measured for the case when the muons are at rest, giving a lifetime τ of approximately $2.2\mu s$. The rate of decay of muons at rest, i.e., in the muon frame, is compared with their rate of decay in flight, i.e., in the Earth frame. In [9] high-velocity muons are used, which causes that the fractional energy loss of the muons in the atmosphere is negligible, making it a constant velocity problem, while in [10] one deals with a variable velocity problem. The discussion of the "muon" experiment in $[I]$ referred to the decay of only one particle. When the real experiments are considered, as are [9] and [10], then we use data on the decay of many such radioactive particles and the characteristic quantities are avareged over many single decay events.

A. The nonrelativistic approach

In the nonrelativistic theory the space and time are separated. The coordinate transformations connecting the Earth frame and the muon frame are the Galilean transformations giving that t_E, the travel time from the mountain to sea level when measured in the Earth frame, is the same as t_μ, which is the elapsed time for the same travelling but measured in the moving frame of the muon, $t_E = t_\mu$. Also, in the nonrelativistic theory, the lifetimes of muons in the mentioned two frames are equal, $\tau_E = \tau_\mu = \tau$. Muon counts on the mountain N_m, and at sea level N_s, as experimentally determined numbers, must not depend on the frame in which they are measured and on the chosen coordinatization. This result, i.e., that $N_{s\mu}$=$N_{sE} = N_s$ and $N_{m\mu} = N_{mE} = N_m$, has to be obtained not only in the nonrelativistic theory but also in the "AT relativity" and in the "TT relativity." The differential equation for the radioctive-decay processes in the nonrelativistic theory can be written as

$$dN/dt = -\lambda N, \quad N_s = N_m \exp(-t/\tau). \tag{1}$$

The travel time t_E is not directly measured by clocks, than, in the Earth frame, it is determined as the ratio of the height of the mountain H_E and the velocity of the muons v, $t_E = H_E/v$. The equation (1) holds in the Earth frame and in the muon frame too, since the two frames are connected by the Galilean transformations, and, as mentioned above, the corresponding times are equal, $t_E = t_\mu$ and $\tau_E = \tau_\mu$. Hence we conclude that in the nonrelativistic theory the exponential factors are the same in both frames and consequently the corresponding fluxes in the two frames are equal, $N_{s\mu}$=N_{sE} and $N_{m\mu} = N_{mE}$, as it must be. However the experiments show that the actual flux at sea level is much higher than that expected from such a nonrelativistic calculation, and thus the nonrelativistic theory does not agree with the experimental results.

B. The "AT relativity" approach

In the "AT relativity" different physical phenomena in different IFRs must be invoked to explain the measured values of the fluxes; the time "dilatation" is used in the Earth frame, but in the muon frame one explains the data by means of the Lorentz "contraction." In order to exploit the results of Secs. 3.2 and 4.2 in [I] we analyse the "muon" experiment not only in the "e" coordinatization but also in the "r" coordinatization. As shown in Sec.4 in [I] the "AT relativity" considers that the spatial and temporal parts of the spacetime length are well-defined physical quantities in 4D spacetime. (But, of course, Sec.4 in [I] also reveals that such an assumption holds only in the Einstein coordinatization, i.e., in the "e" base, see [I].)

Then, as in the nonrelativistic theory, the equation for the radioactive-decay in the "AT relativity" can be written as

$$dN/dx^0 = -\lambda N, \quad N_s = N_m \exp(-\lambda x^0). \tag{2}$$

The equation (2) contains a specific coordinate, the x^0 coordinate, which means that the equation (2) will not remain unchanged upon the Lorentz transformation, i.e., it will not have the same form in different IFRs (and also in different coordinatizations). But in the "AT relativity" it is not required that the physical quantities must be the 4D tensor quantities that correctly transform upon the Lorentz transformations. Thus the quantities in (2) are not the 4D tensor quantities, which actually causes that different phenomena in different IFRs have to be invoked to explain the same physical effect, i.e., the same experimental data. In the Earth frame and in the "e" base we can write in (2) that $x_E^0 = ct_E$, $\lambda_E = 1/c\tau_E$, which gives that the radioactive-decay law becomes $N_{sE} = N_{mE}\exp(-t_E/\tau_E)$. In the experiments [9] and [10] N_{sE}, N_{mE}, and $t_E = H_E/v$ are measured in the Earth frame (tacitly assuming the "e" coordinatization). However the lifetime of muons is measured in their rest frame. Now, in contrast to the nonrelativistic theory where $\tau_E = \tau_\mu$ and $t_E = t_\mu$, the "AT relativity" assumes that in the "e" base there is the time "dilatation" determined by Eq.(20) in Sec.4.2 in [I], which gives the connection between the lifetimes of muons in the Earth frame τ_E and the measured lifetime in the muon frame τ_μ as

$$\tau_E = \gamma\tau_\mu. \tag{3}$$

Using that relation one finds that the radioactive-decay law, when expressed in terms of the measured quantities, becomes

$$N_{sE} = N_{mE}\exp(-t_E/\tau_E) = N_{mE}\exp(-t_E/\gamma\tau_\mu). \tag{4}$$

This equation is used in [9] to make the "relativistic" calculation and compare it with the experimental data. In fact, in [9], the comparison is made between the predicted time dilatation factor γ of the muons and an observed γ. The predicted γ is 8.4 ± 2, while the observed γ is found to be 8.8 ± 0.8, which is a convincing agreement. The prediction of γ is made from the measured energies of muons on the mountain and at sea level; these energies are determined from the measured amount of material which muons penetrated when stopped, and then the energies are converted to the speeds of the muons using the relativistic relation between the total energy and the speed. The observed γ is determined

from the relation (4), where the measured rates were $N_{sE} = 397 \pm 9$ and $N_{mE} = 550 \pm 10$, and the measured height of the mountain is $H_E = 1907m$. The lifetime of muons τ_μ in the muon frame is taken as the information from other experiments (in order to obtain more accurate result) and it is $\tau_\mu = 2.211 \cdot 10^{-6}s$. In [10] the relation for the "relativistic" calculation is written as $N_s = N_m \exp(-t_\mu/\tau_\mu) = N_m \exp(-t_E/(\gamma\tau_\mu))$, Eq.(2) in [10], which shows that the time dilatation is taken into account by the relation $t_\mu = t_E/\gamma$ (the same can be concluded from Eqs.(6) and (7) in [10]). For a given measured flux N_{sE} of muons at sea level, $N_{sE} = 95 \pm 10$, the expected flux on the mountain is determined from a nonrelativistic calculation, Eq.(1), and from a "relativistic" calculation, Eq.(4), i.e., their Eq.(2). The comparison is made between these expected fluxes and the measured counts on the mountain. The predicted counts on the mountain ignoring time dilatation (from (1)) $= 330 \pm 60$; predicted counts on the mountain taking into account time dilatation (from (4)) $= 190 \pm 20$; measured counts on the mountain = 183, and different error bars are reported for this measured counts. We see that the nonrelativistic calculation does not agree with the experimentally found numbers, while the "AT relativity" calculation (made in the "e" base) shows a very convincing agreement with measured fluxes.

Let us now see how the experiments are interpreted in the muon frame. (We note that both [9] and [10] compared the theory (the "AT relativity") and the experiments only in the Earth frame, but using τ_μ from the muon frame.) First we have to find the form of the law for the radioactive-decay processes (2) in the muon frame. As considered above the radioactive-decay law $N_{sE} = N_{mE} \exp(-t_E/\tau_E)$ in the Earth frame and in the "e" base is obtained from the equation (2) using the relations $x_E^0 = ct_E$ and $\lambda_E = 1/c\tau_E$. But, as already said, the equation (2) does not remain unchanged upon the Lorentz transformation and accordingly it cannot have the same form in the Earth frame and in the muon frame. So, actually, in the 4D spacetime, the equation for the radioactive-decay processes in the muon frame could have, in principle, a different functional form than the equation (4), which describes the same radioactive- decay processes in the Earth frame. However, in the "AT relativity," despite of the fact that the quantities in the Earth frame and in the muon frame are not connected by the Lorentz transformations, the equation for the radioactive-decay processes in the muon frame is obtained from the equation (2) in the same way as in the Earth frame, i.e., writting that $x_\mu^0 = ct_\mu$, and $\lambda_\mu = 1/c\tau_\mu$, (as seen in Eq.(2) in [10], the relation for the "relativistic" calculation), whence

$$N_{s\mu} = N_{m\mu} \exp(-t_\mu/\tau_\mu). \tag{5}$$

The justification for such a procedure can be done in the following way. In the "AT relativity" the principle of relativity acts as some sort of "Deus ex machina," which resolves problems; the relation (2) is *proclaimed* to be the physical law and the principle of relativity requires that a physical law must have the same form in different IFRs. (This is the usual way in which the principle of relativity is understood in the "AT relativity.") Therefore, one can write in the equation (2) that $x_E^0 = ct_E$ and $\lambda_E = 1/c\tau_E$ in the Earth frame and $x_\mu^0 = ct_\mu$, and $\lambda_\mu = 1/c\tau_\mu$ in the muon frame. With such substitutions the form of the law is the same in both frames, as it is required by the principle of relativity. Then, as we have already seen, when the consideration is done in the Earth frame, the relation (3) for the time dilatation is used to connect quantities in two frames,.instead of to connect them by the Lorentz transformations. When the consideration is performed in the muon frame another

relation is invoked to connect quantities in two frames. Namely it is considered in the "AT relativity" that in the muon frame the mountain is moving and the muon "sees" the height of the mountain Lorentz contracted,

$$H_\mu = H_E/\gamma, \tag{6}$$

which is Eq.(18), Sec.4.1 in $[I]$, for the Lorentz contraction, giving that

$$t_\mu = H_\mu/v = H_E/\gamma v = t_E/\gamma. \tag{7}$$

This leads to the same exponential factor in Eq.(5) as that one in the Earth frame in Eq.(4), $\exp(-t_\mu/\tau_\mu) = \exp(-t_E/(\gamma\tau_\mu))$. From that result it is concluded that in the "AT relativity" and in the "e" base the corresponding fluxes are equal in the two frames, $N_{s\mu}{=}N_{sE} = N_s$ and $N_{m\mu} = N_{mE} = N_m$. Strictly speaking, it is not the mentioned equality of fluxes, but the equality of ratios of fluxes, $N_{sE}/N_{mE} = N_{s\mu}/N_{m\mu}$, which follows from the equality of the exponential factors in (4) and (5). Both [9] and [10] compared the theory (the "AT relativity") and the experiments only in the Earth frame, but using τ_μ from the muon frame. In [9] the time t_μ that the muons spent in flight according to their own clocks was inferred from the measured distribution of decay times of muons at rest, and in [10] t_E and t_μ are calculated by a simple computer program using the known relation for the mean rate of energy loss of the muons as they travel from the mountain to the sea level and dissipate their energy in the medium (such program is necessary since in [10] one deals with a variable velocity problem).) Since the predicted fluxes N_{sE} and N_{mE} are in a satisfactory agreement with the measured ones, and since the theory (which deals with the time dilatation and the Lorentz contraction) predicts their independence on the chosen frame, it is generally accepted that the "AT relativity" correctly explains the measured data.

The above comparison is worked out only in the "e" coordinatization, but the physics demands that the independence of the fluxes on the chosen frame must hold in all coordinatizations. Therefore we now discuss the experiments [9] and [10] from the point of view of the "AT relativity" but in the "r" coordinatization, see $[I]$. Then, using Eq.(2), we can write the relation for the fluxes in the "r" base and in the Earth frame as

$$N_{r,sE} = N_{r,mE}\exp(-\lambda_{r,E}x^0_{r,E}) = N_{r,mE}\exp(-x^0_{r,E}/x^0_{r,E}(\tau_E)),$$

where $x^0_{r,E}(\tau_E) = 1/\lambda_{r,E}$. Again, as in the "e" base, we have to express $x^0_{r,E}(\tau_E)$ in the Earth frame in terms of the measured quantity $x^0_{r,\mu}(\tau_\mu)$ using the relation (21) from $[I]$ for the time "dilatation" in the "r" base,

$$x^0_{r,E}(\tau_E) = (1+2\beta_r)^{1/2}c\tau_\mu.$$

Hence, the radioactive-decay law (2), in the "r" base, and when expressed in terms of the measured quantities, becomes

$$N_{r,sE} = N_{r,mE}\exp(-x^0_{r,E}/(1+2\beta_r)^{1/2}c\tau_\mu), \tag{8}$$

and it corresponds to the relation (4) in the "e" base. If we express β_r in terms of $\beta = v/c$ as $\beta_r = \beta/(1-\beta)$ (see $[I]$), and use Eq.(8) from $[I]$ to connect the "r" and "e" bases, $x^0_{r,E} = x^0_E - x^1_E = ct_E - H_E$, then the exponential factor in Eq.(8) becomes

$= \exp\left\{-(ct_E - H_E)/\left[(1+\beta)/(1-\beta)\right]^{1/2} c\tau_\mu\right\}$. Using $H_E = vt_E$ this exponential factor can be written in the form that resembles to that one in (4), i.e., it is $= \exp(-t_E/\Gamma_{rE}\tau_\mu)$, and Eq.(8) can be written as

$$N_{r,sE} = N_{r,mE} \exp(-t_E/\Gamma_{rE}\tau_\mu). \tag{9}$$

We see that $\gamma = (1-\beta)^{-1/2}$ in (4) (the "e" base) is replaced by a different factor

$$\Gamma_{rE} = (1+\beta)^{1/2}(1-\beta)^{-3/2} = (1+\beta)(1-\beta)^{-1}\gamma \tag{10}$$

in (9) (the "r" base). The observed Γ_{rE} in the experiments [9] must remain the same, $= 8.8 \pm 0.8$, (it is determined from (9) with the measured values of $N_{r,sE}, N_{r,mE}, t_E$ and τ_μ), but the predicted Γ_{rE}, using the above relation for Γ_r and the known, predicted, $\gamma = 8.4 \pm 2$, becomes $\simeq 250\gamma$,

$$\Gamma_{rE} \simeq 250\gamma. \tag{11}$$

We see that from the common point of view a quite unexpected result is obtained in the "r" coordinatization; the observed Γ_{rE} is as before $= 8.8$, while the predicted Γ_{rE} is $\simeq 250 \cdot 8.4 = 2100$. Similarly, one can show that there is a great discrepancy between the fluxes measured in [9] and [10] and the fluxes predicted when the "dilatation" of time is taken into account but in the "r" coordinatization. Furthermore, it can be easily proved that predicted values in the "r" base and in the muon frame will again greatly differ from the measured ones. *Such results explicitly show that the "AT relativity" is not a satisfactory relativistic theory; it predicts, e.g., different values of the flux N_s (for the same measured N_m) in different synchronizations and for some synchronizations these predicted values are quite different than the measured ones.* These results are directly contrary to the generally accepted opinion about the validity of the "AT relativity."

C. The "TT relativity" approach

Let us now examine the experiments [9] and [10] from the point of view of the "TT relativity." In the "TT relativity" all quantities entering into physical laws must be 4D tensor quantities, and thus with correct transformation properties; *the same 4D quantity* has to be considered in different IFRs and different coordinatizations. In the usual, "AT relativity," analysis of the "muon" experiment, for example, the lifetimes τ_E and τ_μ are considered as the same quantity. Although the transformation connecting τ_E and τ_μ (the dilatation of time, Eq.(3)) is only *a part* of the Lorentz transformation written in the "e" base, it is believed by all proponents of the "AT relativity" that τ_E and τ_μ refer to the same temporal distance (the same quantity) but measured by the observers in two relatively moving IFRs. However, as shown in the preceding sections and in [*I*] ,see Fig.4, in 4D spacetime τ_E and τ_μ refer to different quantities, which are not connected by the Lorentz transformation. To paraphrase Gamba [6]: "As far as relativity is concerned, quantities like τ_E and τ_μ are different quantities, not necessarily related to one another. To ask the relation between τ_E and τ_μ from the point of view of relativity, is like asking what is the relation between the measurement of the radius of the Earth made by an observer S and the

measurement of the radius of Venus made by an observer S'. We can certainly take the ratio of the two measures; what is wrong is the tacit assumption that relativity has something to do with the problem just because the measurements were made by *two* observers."

Hence, in the "TT relativity," instead of the equation (2), which contains x^0 coordinate, we formulate the radioactive-decay law in terms of covariantly defined quantities

$$dN/dl = -\lambda N, \quad N = N_0 \exp(-\lambda l). \tag{12}$$

l is the spacetime length (defined by Eq.(2) in [I]; in the abstract index notation $l = (l^a g_{ab} l^b)^{1/2}$, where $l^a(l^b)$ is the distance 4-vector between two events A and B, $l^a = l^a_{AB} = x^a_B - x^a_A$, $x^a_{A,B}$ are the position 4-vectors and g_{ab} is the metric tensor) for the events of creation of muons (here on the mountain; we denote it as the event O) and their arrival (here at sea level; the event A). $\lambda = 1/l(\tau)$; $l(\tau)$ is the spacetime length for the events of creation of muons (here on the mountain; the event O) and their decay after the lifetime τ, the event T. l, defined in such a way, i.e., as a geometrical quantity, is invariant upon the covariant 4D Lorentz transformations (Eq.(3) in [I]);

$$L^a{}_b \equiv L^a{}_b(v) = g^a{}_b - \frac{2u^a v_b}{c^2} + \frac{(u^a + v^a)(u_b + v_b)}{c^2 - u \cdot v},$$

where u^a is the proper velocity 4-vector of a frame S with respect to itself and v^a is the proper velocity 4-vector of S' relative to S) and, as l is written in the abstract index notation, it does not depend on the chosen coordinatization in the considered IFR. Then in the "e" base and in the muon frame the distance 4-vector l^a_{OA} becomes $l^\alpha_{\mu,OA} = (ct_\mu, 0)$ (the subscript μ will be used, as previously in this section, to denote the quantities in the muon frame, while Greek indices α, β denote the components of some geometric object, e.g., the components $l^\alpha_{\mu,OA}$ in the muon frame of the distance 4-vector l^a_{OA}, see [I] for the notation) and the spacetime length l between these events is $l_{OA} = (l^\beta_{\mu,OA} l_{\mu,\beta OA})^{1/2} = (-c^2 t^2_\mu)^{1/2}$. The representation of the distance 4-vector l^a_{OT} in the "e" base and in the muon frame is $l^\alpha_{\mu,OT} = (c\tau_\mu, 0)$, whence the spacetime length $l_{OT} = (l^\beta_{\mu,OT} l_{\mu,\beta OT})^{1/2} = (-c^2 \tau^2_\mu)^{1/2}$. Inserting the spacetime lengths l_{OA} and l_{OT} into the equation (12) we find the expression for the radioactive-decay law in the "TT relativity"

$$N_s = N_m \exp(-l_{OA}/l_{OT}), \tag{13}$$

which in the "e" base and in the muon frame takes the same form as the relation (5) (the radioactive-decay law in the "AT relativity" in the "e" base and in the muon frame),

$$N_s = N_m \exp(-l_{OA}/l_{OT}) = N_m \exp(-t_\mu/\tau_\mu). \tag{14}$$

Since the spacetime length l is independent on the chosen IFR and on the chosen coordinatization the relation (13) holds in the same form in the Earth frame and in the muon frame and in both coordinatizations, the "e" and "r" coordinatizations. Hence we do not need to examine Eq.(13) in the Earth frame, and in the "r" base, but we can simply compare the relation (14) with the experiments.

Thus, taking into account the discussion given at the begining of Sec.4 in [I], we conclude that, in order to check the validity of the "TT relativity" in the "muon" experiment, we

would need, strictly speaking, to measure, e.g., the lifetime τ_μ and the time t_μ in the muon frame, where they determine l_{OT} and l_{OA} respectively, and then to measure *the same events* (that determined τ_μ and t_μ in the muon frame) in an IFR that is in uniform motion relative to the muon frame (at us it is the Earth frame). Of course it is not possible to do so in the real "muon" experiment but, nevertheless, in this case we can use the data from experiments [9] and [10] and interpret them as that they were obtained in the way required by the "TT relativity." The reasons for such a conclusion are the identity of microparticles of the same sort, the assumed homogeneity and isotropy of the spacetime, and some other reasons that are actually discussed in [9] (although from another point of view). Here we shall not discuss this, in principle, a very complex question, than we take the measured values of τ_μ, t_μ, N_s and N_m and compare them with the results predicted by the relation (14). In [9] τ_μ is taken to be $\tau_\mu = 2.211\mu s$, $N_s = 397 \pm 9$, $N_m = 550 \pm 10$, but t_μ is not measured than it is estimated from Fig.6(a) in [9] to be $t_\mu = 0.7\mu s$. Inserting the values of τ_μ, t_μ and N_m from [9] (for this simple comparison we take only the mean values without errors) into Eq.(14) we predict that N_s is $N_s = 401$, which is in an excellent agreement with the measured $N_s = 397$. As it is already said, the spacetime length l takes the same value in both frames and both coordinatizations, $l_{e,\mu} = l_{e,E} = l_{r,\mu} = l_{r,E}$. Hence, for the measured $N_m = 550$ and if the distance 4-vectors l_{OA}^α and l_{OT}^α would be measured in the Earth frame, and in both frames in the "r" base, we would find the same $N_s = 401$. This result undoubtedly confirms the consistency and the validity of the "TT relativity." (Note that we cannot compare the experiments [10] with the "TT relativity" since their t_μ, Eq.(7) in [10], is not correctly determined from the "TT relativity" viewpoint.)

The nonrelativistic theory predicts the same value of the exponential factor in both frames, $\exp(-t_E/\tau_E) = \exp(-t_\mu/\tau_\mu)$, *since it deals with the absolute time, i.e., with the Galilean transformations. But, for the measured* N_m *the nonrelativistic theory predicts too small* N_s*. The "AT relativity" correctly predicts the value of* N_s *in both frames but only in the "e" coordinatization, while in the "r" coordinatization the experimental* N_s *and the theoretically predicted* N_s *drastically differ. The "TT relativity" completely agrees with the experiments in all IFRs and all possible coordinatizations. Thus, only the manifestly covariant formulation of the special relativity, i.e., the "TT relativity," as the theory of 4D spacetime with the pseudo-Euclidean geometry, is in a complete agreement with the experiments.*

D. Another time "dilatation" experiments

The same conclusion can be obtained comparing the other particle lifetime measurements, e.g., [11], or for the pion lifetime [12], with all three theories. However, as it is already said, all the mentioned experiments, and not only them but all other too, were designed to test the "AT relativity." Thus in the experiments [11], which preceded to the experiments [9] and [10], the relation similar to (4) is used but with t_E replaced by H_E ($=vt_E$) and τ_E (the lifetime of muons in the Earth frame) replaced by $L = v\tau_E$ (L is the "average range before decay"), and also the connection between the lifetimes (3) ($\tau_E = \gamma\tau_\mu$) is employed. Obviously the *predictions* of the results in the experiments [11] will depend on the chosen synchronization, since they deal with the "AT relativity" and use the radioactive-decay law in the form that contains only a part of the distance 4-vector. The *predictions* obtained by the use of the "TT relativity" will be again independent on the chosen IFR and the chosen coordinatization.

However the comparison of these experiments [11] with the "TT relativity" is difficult since, e.g., they have no data for t_μ. Similarly happens with the experiments reported in [12].

The lifetime measurements of muons in the g-2 experiments [14] are often quoted as the most convincing evidence for the time dilatation, i.e., they are claimed as high-precision evidence for the special relativity. Namely in the literature the evidence for the time dilatation is commonly considered as the evidence for the special relativity. The muon lifetime in flight τ is determined by fitting the experimental decay electron time distribution to the six-parameter phenomenological function describing the normal modulated exponential decay spectrum (their Eq.(1)). Then by the use of the relation $\tau = \gamma\tau_0$ and of τ_0 (our τ_μ), the lifetime at rest (as determined by other workers), they obtained the time-dilatation factor γ, or the kinematical γ. This γ is compared with the corresponding dynamical γ factor ($\gamma = (p/m)dp/dE$), which they called $\overline{\gamma}$ (the average γ value). $\overline{\gamma}$ is determined from the mean rotation frequency $\overline{f}_{rot}$ by the use of the Lorentz force law (the "relativistic" expression); the magnetic field was measured in terms of the proton NMR frequency f_p (for the discussion of $g-2$ experiments within the traditional "AT relativity" see also [15]). Limits of order 10^{-3} in $(\gamma - \overline{\gamma})/\gamma$ at the kinematical $\gamma = 29.3$ were set. In that way they also compared the value of the μ^+ lifetime at rest τ_0^+ (from the other precise measurements) with the value found in their experiment $\tau^+/\overline{\gamma}$, and obtained $(\tau_0^+ - \tau^+/\overline{\gamma})/\tau_0^+ = (2 \pm 9) \times 10^{-4}$, (this is the same comparison as the mentioned comparison of γ with $\overline{\gamma}$). They claimed: "At 95% confidence the fractional difference between τ_0^+ and $\tau^+/\overline{\gamma}$ is in the range $(-1.6 - 2.0) \times 10^{-3}$." and "To date, this is the most accurate test of relativistic time dilation using elementary particles." The objections to the precision of the experiments [14], and the remark that a convincing direct test of special relativity must not assume the validity of special relativity in advance (in the use of the "relativistic" Lorentz force law in the determination of the mean rotation frequency and thus of $\overline{\gamma}$, and τ_0), have been raised in [16]. The discussion of these objections is given in [17].

However, our objections to [14] are of a quite different nature. Firstly, the theoretical relations refer to the "e" coordinatization and, e.g., Eq.(1) in the first paper in [14] cannot be transformed in an appropriate way to the "r" coordinatization in order to compare the "AT relativity" in different coordinatizations with the experiments. If only the exponential factor is considered then this factor is again, as in [9], affected by synchrony choice. Although the time t in that exponential factor may be independent of the chosen synchronization (when t is taken to be the multiple of the mean rotation period T), but τ does not refer to the events that happen at the same spatial point and thus it is synchrony dependent quantity. This means that in the "r" base one cannot use the relation $\tau = \gamma\tau_0$ to find the "dilatation" factor γ, but the relation (21) from $[I]$ for the time "dilatation" in the "r" base, $x_r^0(\tau) = (1 + 2\beta_r)^{1/2} c\tau_0$ must be employed. Hence, the whole comparison of γ with $\overline{\gamma}$ holds only in the "e" base; in another coordinatization the "AT relativity" predicts quite different τ_0 for the same $x^0(\tau)$, which is inferred from the exponential decay spectrum.

Let us now examine the measurements [14] from the point of view of the "TT relativity." But for the "TT relativity" these experiments are incomplete and cannot be compared with the theory. Namely, in the "TT relativity," as already said, it is not possible to find the values of the muon lifetime in flight τ by analyses of the measurements of the radioactive decay distribution, since, there, the radioactive decay law is written in terms of the spacetime lengths and not with t and τ. Also, in the "TT relativity," there is not the connection between

the muon lifetime in flight τ and the lifetime at rest τ_0 in the form $\tau = \gamma\tau_0$, since τ, in the "TT relativity," does not exist as a well defined quantity. Thus, in the "TT relativity," there is no sense in the use of the relation $\tau = \gamma\tau_0$ to determine γ. An important remark is in place here; in principle, in the "TT relativity," the same events and the same quantities have to be considered in different frames of reference, which means that in the muon experiment [14] the lifetime at rest τ_0 refers to the decaying particle in an accelerated frame and for the theoretical discussion we would need to use the coordinate transformations connecting an IFR with an accelerated frame of reference. (An example of the generalized Lorentz transformation is given in [18] but they are written in the "e" base and thus not in fully covariant way, i.e., not in the way as we have written the covariant Lorentz transformation, Eq.(3) in [*I*].) Furthermore, in the experiments [14] the average value of γ ($\overline{\gamma}$), i.e., the dynamical γ, for the circulating muons is found by analysis of the bunch structure of the stored muon and the use of the relation connecting $\overline{\gamma}$ and the mean rotation frequency $\overline{f}_{rot}$; this relation is obtained by the use of the expression for the "relativistic," i.e., the "AT relativity," Lorentz force law, which is expressed by means of the 3-vectors $\mathbf{E}$ and $\mathbf{B}$. However, in contrast to the "AT relativity," and also to the usual covariant formulation, in the "TT relativity," the covariant Lorentz force $K^a = (q/c)F^{ab}u_b$ (F^{ab} is the electromagnetic field tensor and u^b is the 4-velocity of a charge q; all is written in the abstract index notation, [19] and [20]) cannot be expressed in terms of the 3-vectors $\mathbf{E}$ and $\mathbf{B}$. Namely, as already said, in the "AT relativity" the real physical meaning is attributed not to F^{ab} than to the 3-vectors $\mathbf{E}$ and $\mathbf{B}$, while in the "TT relativity" only covariantly defined quantities do have well-defined physical meaning both in the theory and in experiments. (The transformations of the 3-vectors $\mathbf{E}$ and $\mathbf{B}$ are not directly connected with the Lorentz transformations of the *whole 4D tensor quantity* F^{ab} as a geometrical quantity, but indirectly through the transformations of *some components* of F^{ab}, and that, *in the specific coordinatization, the Einstein coordinatization.* This issue is discussed in [1] and [4,21], where it is also shown that the 3-vector $\mathbf{E}$ ($\mathbf{B}$) in an IFR S and the transformed 3-vector $\mathbf{E}'$ ($\mathbf{B}'$) in relatively moving IFR S' do not refer to the same physical quantity in 4D spacetime, i.e., that the conventional transformations of $\mathbf{E}$ and $\mathbf{B}$ are the AT.) From [20] and [4] one can see how the Lorentz force K^a is expressed in terms of the 4-vectors E^a and B^a and show when this form corresponds to the classical expression for the Lorentz force with the 3-vectors $\mathbf{E}$ and $\mathbf{B}$. Also it can be seen from [4] that for $B^\alpha \neq 0$ (B^α is the representation of B^a in the "e" base) it is not possible to obtain $\gamma_u = 1$ (the 4-velocity of a charge q in the "e" base is $u^\alpha = (\gamma_u c, \gamma_u \mathbf{u})$ and $\gamma_u = (1 - u^2/c^2)^{-1/2}$) and the covariant Lorentz force K^a can never take the form of the usual magnetic force $\mathbf{F_B}$. Hence it follows that in the "TT relativity" it is not possible to use the Lorentz force $\mathbf{F_B}$ and the usual equation of motion $d(\overline{\gamma}m\mathbf{u})/\mathbf{dt}{=}\mathbf{q}(\mathbf{u}\times\mathbf{B})$ to find the relation connecting $\overline{\gamma}$ and the mean rotation frequency $\overline{f}_{rot}$, and thus to find τ_0 from $\tau/\overline{\gamma}$, .in the way as in [14]. The discussion about the kinematical γ (the relation $\tau = \gamma\tau_0$) and about the dynamical $\overline{\gamma}$ (from the use of the Lorentz force) shows that the measurements [14] cannot be compared with the "TT relativity." But, as we explained before, in contrast to the usual opinion, these experiments do not confirm the "AT relativity" either, since if the exponential decay spectrum is analyzed in another coordinatization, e.g., the "r" coordinatization, then, similarly as for the experiments [9], one finds that for the given N_0 the theoretical and the experimental N differ.

IV. THE MICHELSON-MORLEY EXPERIMENT

These conclusions will be further supported considering some other experiments, which, customarily, were assumed to confirm the "AT relativity." The first one will be the famous Michelson-Morley experiment [22], and some modern versions of this experiment will be also discussed.

In the Michelson-Morley experiment two light beams emitted by one source are sent, by half-silvered mirror O, in orthogonal directions. These partial beams of light traverse the two equal (of the length L) and perpendicular arms OM_1 (perpendicular to the motion) and OM_2 (in the line of motion) of Michelson's inteferometer and the behaviour of the interference fringes produced on bringing together these two beams after reflection on the mirrors M_1 and M_2 is examined. In order to avoid the influence of the effect that the two lengths of arms are not exactly equal the entire inteferometer is rotated through 90^0. Then any small difference in length becomes unimportant. The experiment consists of looking for a shift of the intereference fringes as the apparatus is rotated. The expected maximum shift in the number of fringes (the measured quantity) on a 90^0 rotation is

$$\triangle N = \triangle(\phi_2 - \phi_1)/2\pi, \tag{15}$$

where $\triangle(\phi_2 - \phi_1)$ is the change in the phase difference when the interferometer is rotated through 90^0. ϕ_1 and ϕ_2 are the phases of waves moving along the paths OM_1O and OM_2O, respectively.

A. The nonrelativistic approach

In the nonrelativistic approach the speed of light in the preferred frame is c. Then, on the ether hypothesis, the speed of light, in the Earth frame, i.e., in the rest frame of the interferometer (the S frame), on the path along an arm of the Michelson interferometer oriented perpendicular to its motion at velocity $\mathbf{v}$ relative to the preferred frame (the ether) is $(c^2 - v^2)^{1/2}$; the Earth together with the inteferometer moving with velocity $\mathbf{v}$ through the ether is equivalent to the inteferometer at rest with the ether streaming through it with velocity $-\mathbf{v}$. Since in S both waves are brought together to the same spatial point the phase difference $\phi_2 - \phi_1$ is determined only by the time difference $t_2 - t_1$; $\phi_2 - \phi_1 = 2\pi(t_2 - t_1)/T$, where t_1 and t_2 are the times required for the complete trips OM_1O and OM_2O, respectively, and $T(=\lambda/c)$ is the period of vibration of the light. From the known speed of light one finds that t_1 is

$$t_1 = 2L/c(1 - v^2/c^2)^{1/2}. \tag{16}$$

Similarly, the speed of light on the path OM_2 is $c - v$, and on the return path is $c + v$, giving that

$$t_2 = 2L/c(1 - v^2/c^2). \tag{17}$$

We see that according to the nonrelativistic approach the time t_1 is a little less than the time t_2, even though the mirrors M_1 and M_2 are equidistant from O. To order v^2/c^2 the difference

in the times is $t_2 - t_1 = (L/c)(v^2/c^2)$. Inserting it into $\triangle N$ (15) (the measured quantity $\triangle N$, when the phase difference is determined by the time difference, is $\triangle N = 2(t_2 - t_1)c/\lambda$) we find, to the same order v^2/c^2, that

$$\triangle\, N \simeq (2L/\lambda)(v^2/c^2). \tag{18}$$

This result is obtained by the classical analysis in the Earth frame (the interferometer rest frame).

Let us now consider the same experiment in the preferred frame (the S' frame).Since in the nonrelativistic theory the two frames are connected by the Galilean transformations, it follows that the corresponding times in both frames are equal, $t_1 = t'_1$ and $t_2 = t'_2$, whence $t_2 - t_1 = t'_2 - t'_1$ and, supposing that again the phase difference is determined only by the time difference, $\triangle N = \triangle N'$. However, for the further purposes, it is worth to find explicitly t'_1 and t'_2 considering the experiment directly in the preferred frame. Since the speed of light in the preferred frame is c, the preferred-frame observer considers that the light travels a distance $ct'_1/2$ along the hypotenuse of a triangle; in the same time $t'_1/2$ the mirror M_1 moves to M'_1, i.e., to the right a distance $vt'_1/2$, and from the right triangle this observer finds $t'_1/2 = L/c(1 - v^2/c^2)^{1/2}$. The return trip is again along the hypotenuse of a triangle and the return time is again $= t'_1/2$. The total time for such a zigzag path is, as it must be, $t'_1 = t_1$ (16) (the half-silvered mirror O moved to O' in t'_1). For the arm oriented parallel to its motion the preferred-frame observer considers that the light, when going from O to M'_2, must traverse a distance $L + vt'_3$ at the speed c, whence $L + vt'_3 = ct'_3$ and $t'_3 = L/(c - v)$. In a like manner, the time t'_4 for the return trip is $t'_4 = L/(c + v)$. The total time $t'_2 = t'_3 + t'_4$ is, as it must be, equal to t_2 (17) (the half-silvered mirror O moved to O'' in t'_2). This discussion shows that the nonrelativistic theory is a consistent theory giving the same $\triangle N$ in both frames. However it does not agree with the experiment. Namely Michelson and Morley found from their experiment that was no observable fringe shift.

From the theoretical point of view it is interesting to mention an analysis of the Michelson-Morley experiment which is given in [23]. There, the paths of light are examined in the case when the experiment is viewed from a frame in which the apparatus is moving at velocity v (our S' frame). It is inquired whether the half-silvered mirror O correctly reflects the light to and from the interferometer arms, such that light travels in the appropriate "triangular path" in the transverse arm, and correctly brings the longitudinal ray into line with the transverse ray at the detector. The result is that if in the classical analysis the half-silvered mirror is set to exactly 45^0, then the transverse ray will "overshoot" the desired trajectory while the longitudinal ray will "undershoot." The interference pattern will be dependent on the position of the detector since there is a divergence of the interfering light rays. The ray angles on the way to the detector are given by Eq.(16) in [23] The difference in these angles is exceedingly small (second order in v/c), and hence negligible in the usual Michelson-Morley experiment.

At this point it has to be noted that there are more serious objections to the traditional derivation of $\triangle N$ in the nonrelativistic theory and in S' than the one mentioned by [23]. These objections are usually overlooked and we only briefly sketch them here. Firstly, in S' the waves are not brought together to the same spatial point and consequently the phase difference is not determined only by the time difference. Strictly speaking the increment of phase ϕ'_1 for the trip OM'_1O' is $\phi'_1 = (\omega'_{OM'_1} t'_1/2 - \mathbf{k'_{OM'_1} l'_{OM'_1}}) + (\omega'_{\mathbf{M'_1O'}} \mathbf{t'_1/2} - \mathbf{k'_{M'_1O'} l'_{M'_1O'}})$,

and similarly the increment of phase ϕ_2' for the trip $OM_2'O''$ is $\phi_2' = (\omega_{OM_2'}' t_3' - \mathbf{k}_{\mathbf{OM_2'}}' \mathbf{l}_{\mathbf{OM_2'}}') + (\omega_{\mathbf{M_2'O''}}' \mathbf{t_4'} - \mathbf{k}_{\mathbf{M_2'O''}}' \mathbf{l}_{\mathbf{M_2'O''}}')$, where $\omega_{OM_1'}'$, $\mathbf{k}_{\mathbf{OM_1'}}'$, and $\mathbf{l}_{\mathbf{OM_1'}}'$ are the angular frequency, the wave 3-vector and the distance 3-vector $(\overrightarrow{OM_1'})$, respectively, of the wave on the trip OM_1', etc.. What is overlooked in the usual derivation of $\triangle N$ in the nonrelativistic theory is that not all ω' are the same due to the classical Doppler effect of inertial motion of a source and of a mirror in the S' frame, and that the classical aberration of light has to be taken into account when different $\mathbf{k}'$ in S' are determined (this is, in fact, considered in [23]). We shall not examine the mentioned changes of the classical derivation since $\triangle N$, obtained with these changes, will be again different from zero.

It is possible to look at the Michelson-Morley experiment from another point of view; the light ray going both ways in one of the arms of the interferometer can be considered as a clock, a light clock, with the period determined by the return time of the light ray. The experiment is then considered as the comparison of the frequencies of two clocks, and it shows that the relative frequency does not change by a rotation of the interferometer. Such point of view is important for the interpretation of the modern versions of the Michelson-Morley experiment.

B. The "AT relativity" approach

Next we examine the same experiment from the "AT relativity" viewpoint. Again, as in the discussion of the "muon" experiment, we consider this experiment in both frames and in both coordinatizations as well. First the "e" coordinatization in both frames will be explored. It has to be noted that the experiment is usually discussed only in the "e" base, and again, as in the nonrelativistic theory, the phase difference $\phi_2 - \phi_1$ is considered to be determined only by the time difference $t_2 - t_1$. Further, in contrast to the nonrelativistic theory, in the "AT relativity" and in the "e" base it is postulated (Einstein's second postulate) that light *always* travels with speed c.

Hence in the S frame (the rest frame of the interferometer) $t_1 = 2L/c$ and $t_2 = 2L/c$, and, with the assumption that only the time difference $t_2 - t_1$ matters, it follows that $\triangle N$=0, in agreement with the experiment. In the S' frame (the preferred frame) the time t_1' is determined in the same way as in the nonrelativistic theory, i.e., supposing that a zigzag path is taken by the light beam in a moving "light clock". Thus, the light-travel time t_1' is exactly equal to that one in the nonrelativistic theory, $t_1' = 2L/c(1 - v^2/c^2)^{1/2}$. Comparing with $t_1 = 2L/c$ we see that, in contrast to the nonrelativistic theory, it takes a longer time for light to go from end to end in the moving clock than in the stationary clock, $t_1' = t_1/(1 - v^2/c^2)^{1/2} = \gamma t_1$, see, e.g., [7] p.15-6, [8] p.359, or an often cited paper on modern tests of special relativity [13]. This is the usual way in which it is shown how, in the "AT relativity," the time dilatation is forced upon us by the constancy of the speed of light. However, in the "AT relativity," the light-travel time t_2' is determined by invoking the Lorentz contraction; it is argued that a preferred frame observer measures the length of the arm oriented parallel to its motion to be contracted to a length $L' = L(1 - v^2/c^2)^{1/2}$, see, e.g., [13]. Then t_2' is determined in the same way as in the nonrelativistic theory but with L' replacing the rest length L, $t_2' = (L'/(c-v)) + (L'/(c+v)) = 2L/c(1-v^2/c^2)^{1/2} = t_1'$, whence $t_2' - t_1' = 0$ and $\triangle N'$=0, as in the S frame. We quoted such usual derivation in order to

illustrate how the time dilatation and the Lorentz contraction are used in the "AT relativity" to show the agreement between the theory and the famous Michelson-Morley experiment. Although this procedure is generally accepted by the majority of physicists as the correct one and quoted in all textbooks on the subject, we note that such an explanation of the null result of the experiment is very awkward and does not use at all the 4D symmetry of the spacetime; the derivation deals with the temporal and spatial distances as well defined quantities, i.e., in a similar way as in the prerelativistic physics, and then in an artificial way introduces the changes in these distances due to the motion.

To better illustrate the preceding assertions we derive the same results for t_1', t_2' and $\triangle N'$ in another way too. It starts with 4D quantities, but then, as shown in $[I]$ Sec.4.2, connects only some parts of the distance 4-vectors, i.e., the temporal distances, in two relatively moving frames using Eq.(20) from $[I]$ for the time dilatation in the "e" base instead of the complete Lorentz transformation. Let now A, B and A_1 denote the events; the departure of the transverse ray from the half-silvered mirror O, the reflection of this ray on the mirror M_1 and the arrival of this beam of light after the round trip on the half-silvered mirror O, respectively. In the same way we have, for the longitudinal arm of the inteferometer, the corresponding events A, C and A_2. Then, from Eq.(20) in $[I]$, one finds $t_1' = \gamma t_1$, $t_2' = \gamma t_2 = t_1'$ and consequently $\triangle N'$=0. But, note, that in both mentioned derivations in the "AT relativity," the fequencies of the waves are supposed to be the same in S and S', i.e., the Doppler effect is not taken into account, and the contributions of $\mathbf{k}'$ and $\mathbf{l}'$ to the increments of phase in S' are neglected, i.e., the consideration of the aberration of light in the determination of different $\mathbf{k}'$ in S' is not performed. Obviously, in the "AT relativity," the same procedure is applied to the calculation of ϕ_1', ϕ_2' and $\triangle N'$ in S' as in the nonrelativistic theory; only, in an artificial way, the Lorentz contraction and the time dilatation are introduced into the calculation.

The same experiment can be examined in the "r" coordinatization**.** This synchronization is an asymmetric synchronization, which leads to an asymmetry in the measured one-way speed of light, but the average speed of light on any round trip is independent of the synchronization procedure employed, and is $= c$. In the Michelson-Morley experiment the measured phase difference between the phases on the round trips OM_1O and OM_2O in S, the rest frame of the interferometer, is synchrony independent, since both waves are brought together to the same spatial point. Hence, one concludes that the same result $\triangle N$=0 will be obtained in the S frame in the "r" base as in the "e" coordinatization. However in the S' frame such independence on the used coordinatization cannot be expected. We shall not discuss this issue here for the sake of saving space, and since there are some more important problems in the traditional "AT relativity" derivation of $\triangle N$.

As already mentioned, it is shown in [23] that the classical analysis of the interference, in the frame in which the apparatus is in motion, predicts a divergence of the interfering light rays on the way to the detector. In contrast to this result, the exact parallelness of the longitudinal and the transverse rays is obtained in [23], but only in the case when, in addition to the usual analysis in the "AT relativity," the Lorentz contraction "tilt" of the moving half-silvered mirror is taken into account. The analysis in [23] actually takes into account the aberration of light, which is overlooked in the traditional derivation in the "AT relativity" in the same way as it is overlooked in the nonrelativistic theory. However this analysis is performed in the "e" coordinatization and the Lorentz contraction "tilt" of the moving

half-silvered mirror, that is required for the exact parallelness of the rays, is considered in that coordinatization. In another coordinatization, e.g., in the "r" coordinatization, the Lorentz "contraction" of the moving half-silvered mirror will be different, it can become a "dilatation," and one can expect a divergence of the interfering light rays on the way to the detector. But, as it is already said, the effect is exceedingly small (second order in v/c), and hence negligible in the usual Michelson-Morley experiment, and therefore it will not be discussed in more detail.

1. Driscoll's non-null fringe shift

In [24] the usual "AT relativity" calculation in the "e" base (see the discussion above and also [7], [8], [13]) of the fring shift in the Michelson-Morley experiment is repeated, and, of course, the observed null fringe shift independent of changes of v, the relative velocity of S and S', and/or θ, the angle that the undivided ray from the source to the beam divider makes with $\mathbf{v}$, is obtained. However, it is noticed in [24] that in such a traditional calculation of $\triangle(\phi_2-\phi_1)$ (the change in the phase difference when the interferometer is rotated through 90^0) only path lengths (optical or geometrical), i.e., the temporal distances, are considered, while the Doppler effect on wavelength in the S' frame, in which the interferometer is moving, is not taken into account. Then the same calculation of $\triangle(\phi_2'-\phi_1')$, as the traditional one, is performed in [24], but determing the increment of phase along some path, e.g. OM_1', not only by the segment of geometric path length (i.e., the temporal distance for that path) than also by the wavelength in that segment (i.e., the frequency of the wave in that segment). Accordingly the phase difference (in our notation) $\phi_1'-\phi_2'$, in the S' frame, between the ray along the vertical path $OM_1'O'$, and that one along the longitudinal path $OM_2'O''$, respectively, is found (see [24]) to be

$$(\phi_1'-\phi_2')_{(b)}/2\pi = 2(L\nu/c)(1+\varepsilon+\beta^2)-2(L\nu/c)(1+2\beta^2) = 2(L\nu/c)(\varepsilon-\beta^2), \qquad (19)$$

Eqs.(23-25) in [24], where L is the length of the segment OM_2 and $\overline{L}=L(1+\varepsilon)$ is the length of the arm OM_1 (L, $\overline{L}$ and ν are determined in the rest frame of the interferometer). In this expression the Doppler effect of $\mathbf{v}$ on the frequencies, and the Lorentz contraction of the longitudinal arm, are taken into account. In a like manner Driscoll finds the phase difference in the case when the interferometer is rotated through 90^0

$$(\phi_1'-\phi_2')_{(a)}/2\pi = 2(L\nu/c)(1+\varepsilon+2\beta^2)-2(L\nu/c)(1+\beta^2) = 2(L\nu/c)(\varepsilon+\beta^2), \qquad (20)$$

Eqs.(19-21) in [24]. Hence, it is found in [24] a "surprising" non-null fringe shift

$$\triangle N' = \triangle(\phi_2'-\phi_1')/2\pi = 4(L\nu/c)\beta^2, \qquad (21)$$

where $\triangle(\phi_2'-\phi_1') = (\phi_1'-\phi_2')_{(b)}-(\phi_1'-\phi_2')_{(a)}$, and we see that the entire fringe shift is due to the Doppler shift. From the non-null result (21) the author of [24] concluded: "that the Maxwell-Einstein electromagnetic equations and special relativity jointly are disproved, not confirmed, by the Michelson-Morley experiment." However such a conclusion cannot be drawn from the result (21). The origin of the appearance of $\triangle N'\neq 0$ (21) is quite different than that considered in [24], and it will be explained below. Note that in [23] the changes

in the usual derivation of $\triangle N'$, which are caused by the aberration of light, are considered, while [24] investigates those changes which are caused by the Doppler effect. Both changes are examined only in the "e" base, and both would be different in, e.g., the "r" base. This means that $\triangle N'$ in S' will be dependent on the chosen synchronization, and consequently that the "AT relativity" is not capable to explain in a satisfactory manner the results of the Michelson-Morley experiment.

C. The "TT relativity" approach

Next we examine the Michelson-Morley experiment from the "TT relativity" viewpoint. Then the relevant quantity is the phase of a light wave, and it is (when written in the abstract index notation, see [*I*] and [19])

$$\phi = k^a g_{ab} l^b, \tag{22}$$

where k^a is the propagation 4-vector, g_{ab} is the metric tensor and l^b is the distance 4-vector. (We note that in the "TT relativity" the light waves are described by the 4-vectors $E^a(x^b)$ and $B^a(x^b)$ of the electric and magnetic fields ($E^a(x^b) = E_0^a \exp(ik^b x_b)$), while the "AT relativity" works with the 3-vectors $\mathbf{E}(\mathbf{r},\mathbf{t})$ and $\mathbf{B}(\mathbf{r},\mathbf{t})$ ($\mathbf{E}(\mathbf{r},\mathbf{t}) = \mathbf{E_0} \exp(\mathbf{i}(\mathbf{kr}-\omega\mathbf{t}))$), as in the prerelativistic physics.) The traditional derivation of $\triangle N$ (in the "AT relativity") deals only with the calculation of t_1 and t_2 in S and S', but does not take into account either the changes in frequencies due to the Doppler effect or the aberration of light. The "AT relativity" calculations in [24] and [23] improve the traditional procedure taking into account the changes in frequencies [24], and the aberration of light [23], but only in the "e" base. None of the "AT relativity" calculations deals with the covariantly defined 4D quantities, in this case, the covariantly defined phase (22), and it will be shown here that the non-null theoretical result obtained in [24] is a consequence of that fact. In the "TT relativity" neither the Doppler effect nor the aberration of light exist separately as well defined physical phenomena. The separate contributions to ϕ of the ωt factors and $\mathbf{kl}$ factors are, in general case, meaningless in the "TT relativity" and only their indivisible unity, the phase (22), is meaningful quantity; it is invariant upon the covariant 4D Lorentz transformations (Eq.(3) in [*I*] and Sec.3 here), and, as it is written in the abstract index notation, the phase (22) does not depend on the chosen coordinatization in the considered IFR. (All quantities in (22), i.e., k^a, g_{ab} and l^b, are the 4D tensor quantities that correctly transform upon the covariant 4D Lorentz transformations (Eq.(3) in [*I*] and Sec.3 here), which means that in all relatively moving IFRs always *the same 4D quantity*, e.g., k^a, or l^b, is considered. This is not the case in the "AT relativity" where, for example, the relation $t_1' = \gamma t_1$ is not the Lorentz transformation of some 4D quantity, and t_1' *and* t_1 *do not correspond to the same 4D quantity* considered in S' and S respectively.) Only in the "e" coordinatization the ωt and $\mathbf{kl}$ factors can be considered separately. Therefore, and in order to retain the similarity with the prerelativistic and the "AT relativity" considerations, we first determine ϕ (22) in the "e" base and in the S frame (the rest frame of the interferometer). Then k_{ABe}^μ and l_{ABe}^μ (the representations of k_{AB}^a and l_{AB}^a in the "e" base and in S) for the wave on the trip OM_1 (A and B are the corresponding events for that trip, as mentioned previously) $k_{ABe}^\mu = (\omega/c, 0, 2\pi/\lambda, 0)$ and $l_{ABe}^\mu = (ct_{M_1}, 0, \overline{L}, 0)$, while for the wave on the return trip M_1O,

(the events B and A_1) $k^{\mu}_{BA_1e} = (\omega/c, 0, -2\pi/\lambda, 0)$ and $l^{\mu}_{BA_1e} = (ct_{M_1}, 0, -\overline{L}, 0))$. Hence the increment of phase ϕ_{1e}, for the the round trip OM_1O, is

$$\phi_{1e} = k^{\mu}_{ABe} l_{\mu ABe} + k^{\mu}_{BA_1e} l_{\mu BA_1e} = 2(-\omega t_{M_1} + (2\pi/\lambda)\overline{L}),$$

where ω is the angular frequency and, for the sake of comparison with [24], the length of the arm OM_1 is taken to be $\overline{L} = L(1+\varepsilon)$, and L is the length of the segment OM_2. In a like manner we find k^{μ}_{ACe} and l^{μ}_{ACe} for the wave on the trip OM_2, (the corresponding events for the round trip OM_2O are A, C and A_2) as $k^{\mu}_{ACe} = (\omega/c, 2\pi/\lambda, 0, 0)$ and $l^{\mu}_{ACe} = (ct_{M_2}, L, 0, 0)$, while for the wave on the return trip M_2O, $k^{\mu}_{CA_2e} = (\omega/c, -2\pi/\lambda, 0, 0)$ and $l^{\mu}_{CA_2e} = (ct_{M_2}, -L, 0, 0))$, whence

$$\phi_{2e} = k^{\mu}_{ACe} l_{\mu ACe} + k^{\mu}_{CA_2e} l_{\mu CA_2e} = 2(-\omega t_{M_2} + (2\pi/\lambda)L),$$

and thus

$$\phi_{1e} - \phi_{2e} = -2\omega(t_{M_1} - t_{M_2}) + 2(2\pi/\lambda)(\overline{L} - L). \tag{23}$$

Particularly for $\overline{L} = L$, and consequently $t_{M_1} = t_{M_2}$, one finds $\phi_{1e} - \phi_{2e} = 0$. It can be easily shown that the same difference of phase (23) is obtained in the case when the interferometer is rotated through 90^0, whence we find that $\triangle(\phi_{1e} - \phi_{2e})$=0, and $\triangle N_e = 0$. Since, according to the construction, ϕ (22) is a Lorentz scalar, and does not depend on the chosen coordinatization in a considered IFR, we immediately conclude, without calculation, that

$$\triangle N'_e = \triangle N_r = \triangle N'_r = \triangle N_e = 0, \tag{24}$$

which is in a complete agreement with the Michelson-Morley experiment.

1. *Explanation of Driscoll's non-null fringe shift and of the null fringe shift obtained in the conventional "AT relativity" calculation*

Driscoll's improvement of the traditional "AT relativity" derivation of the fringe shift can be easily obtained from our covariant approach taking only the product $k'^0_e l'_{0e}$ in the calculation of the increment of phase ϕ'_e in S' in which the apparatus is moving. Thus in S' $k'^{\mu}_{ABe} = (\gamma\omega/c, -\beta\gamma\omega/c, 2\pi/\lambda, 0)$ and $l'^{\mu}_{ABe} = (\gamma ct_{M_1}, -\beta\gamma ct_{M_1}, \overline{L}, 0)$, and also $k'^{\mu}_{BA_1e} = (\gamma\omega/c, -\beta\gamma\omega/c, -2\pi/\lambda, 0)$ and $l'^{\mu}_{BA_1e} = (\gamma ct_{M_1}, -\beta\gamma ct_{M_1}, -\overline{L}, 0)$, giving that

$$(-1/2\pi)(k'^0_{ABe} l'_{0ABe} + k'^0_{BA_1e} l'_{0BA_1e}) = 2\gamma^2 \nu t_{M_1} \simeq 2(L\nu/c)(1 + \varepsilon + \beta^2), \tag{25}$$

which is exactly equal to Driscoll's result $\triangle P_{Hb}$, for our notation see (19). In a like manner one finds that

$$(-1/2\pi)(k'^0_{ACe} l'_{0ACe} + k'^0_{CA_2e} l'_{0CA_2e}) = 2\gamma^2(\nu t_{M_2} + \beta^2 L/\lambda) \simeq 2(L\nu/c)(1 + 2\beta^2), \tag{26}$$

which is Driscoll's result $\triangle P_{\Xi b}$, see (19). In the same way we can find in S' Driscoll's result (20) and finally the non-null fringe shift, Eq.(21). The same calculation of $k'^i_e l'_{ie}$, the contribution of the spatial parts of k'^{μ}_e and $l'_{\mu e}$ to $\triangle N'_e$, shows that this term exactly cancel

the $k_e^{\prime 0} l'_{0e}$ contribution (Driscoll's non-null fringe shift (21)), yielding that $\triangle N'_e = 0$. We note that the calculation in [24] actually assumes that $k_e^0 l_{0e}$ and $k_e^{\prime 0} l'_{0e}$ refer to the same quantity measured by the observers in S and S'; of course, it is supposed that S and S' are connected by the Lorentz transformation, and consequently that the quantities $k_e^0 l_{0e}$ and $k_e^{\prime 0} l'_{0e}$ are connected by the Lorentz transformation as well. However the relations (25) and (26) are not the Lorentz transformation of some 4D quantity, and really *$k_e^0 l_{0e}$ and $k_e^{\prime 0} l'_{0e}$ do not refer to the same 4D quantity* considered in S and S' respectively. Thus in this case too the quantities $k_e^0 l_{0e}$ and $k_e^{\prime 0} l'_{0e}$ are connected by the AT, and, as Gamba [6] says, whenever two quantities, which are connected by the AT, are considered to refer to the same physical quantity in 4D spacetime we have *the case of mistaken identity.*

The traditional "AT relativity" analysis of the experiment deals only with the calculation of t_1 and t_2 in S and S', (t_1 in S is $= 2t_{M_1}$ and ct_{M_1} is only zeroth component l_{ABe}^0 $(= ct_{M_1})$ of l_{ABe}^μ, and similarly for t_2 and ct_{M_2}). Furthermore, such traditional "AT relativity" calculation simply connects, e.g., zeroth component l_{ABe}^0 in S and $l_{ABe}^{\prime 0}$ in S', or l_{ACe}^0 and $l_{ACe}^{\prime 0}$, etc., by the relation for the time "dilatation" in the "e" base, $l_{ABe}^{\prime 0} = \gamma ct_{M_1}$, and also $l_{ACe}^{\prime 0} = \gamma ct_{M_2}$, although $l_{ACe}^{\prime 0}$, when determined by the Lorentz transformation, is $l_{ACe}^{\prime 0} = \gamma(ct_{M_2} - \beta L)$, which then yields that $t'_1 = \gamma t_1$, $t'_2 = \gamma t_2 = t'_1$, and consequently one finds the null fringe shift $\triangle N'$=0. It is clear from this discussion that, in contrast to the usual opinion, the quantities, e.g., t'_1 and t_1, etc., do not refer to the same quantity, which is measured in relatively moving IFRs S' and S, but they are different 4D quantities that are not connected by the Lorentz transformation. Therefore the agreement of he traditional "AT relativity" calculation with the results of the Michelson-Morley experiment is not the "true" agreement, but an "apparent" agreement that is achieved by an incorrect procedure.

The whole discussion about the the Michelson-Morley experiment reveals that, contrary to the generaly accepted opinion, the Michelson-Morley experiment does not confirm the validity of the traditional Einstein approach to the special relativity, i.e., the "AT relativity," than it confirms the validity of a compete covariant approach, both in the theory and experiments, i.e., it confirms the "TT relavitity."

D. The modern laser versions

The modern laser versions of the Michelson-Morley experiment, e.g., [25] and [26], are always interpreted according to the "AT relativity." They rely on highly monochromatic (maser) laser frequency metrology rather than optical interferometry; the measured quantity is not the maximum shift in the number of fringes than a beat frequency variation and the associated (maser) laser-frequency shift. In [25] the authors recorded the variations in beat frequency between two optical maser oscillators when rotated through 90^0 in space; the two maser cavities are placed orthogonally on a rotating table and they can be considered as two light clocks. It is stated in [25] that the highly monochromatic frequencies of masers; "...allow very sensitive detection of any change in the round-trip optical distance between two reflecting surfaces." and that the comparison of the frequencies of two masers allows: "...a very precise examination of the isotropy of space with respect to light propagation." The result of this experiment was: "... there was no relative variation in the maser frequencies associated with orientation of the earth in space greater than about 3 kc/sec." Similarly [26] compares the frequencies of a He-Ne laser locked to the resonant frequency of a higly

stable Fabry-Perot cavity (the meter-stick, i.e., "etalon of length") and of a CH_4 stabilized "telescope-laser" frequency reference system. The beat frequency of the isolation laser (CH_4 stabilized-laser) with the cavity-stabilized laser was the measured quantity; a beat frequency variation is considered when the direction of the cavity length is rotated. The authors of [26], in the same way as [25], consider their experiment as: "isotropy of space experiment." Namely it is stated in [26] that: "Rotation of the entire electro-optical system maps any cosmic directional anisotropy of space into a corresponding frequency variation." They found a null result, i.e., a fractional length change of $\triangle l/l = (1.5 \pm 2.5) \times 10^{-15}$ (this is also the fractional frequency shift) in showing the isotropy of space; this result represented a 4000-fold improvement on the measurements [25]. In [13] the experiment [26] is quoted as the most precise repetition of the Michelson-Morley experiment, and it is asserted that the experiment [26] constrained the two times, our t_1' and t_2', to be equal within a fractional error of 10^{-15}. The times t_1' and t_2' refer to the round-trips in two maser cavities in [25], and to the round-trips in the Fabry-Perot cavity in [26]. These times are calculated in the same way as in the Michelson-Morley experiment.(see, for example, [13]).

The above brief discussion of the experiments [25] and [26], and the previous analysis of the usual, "AT relativity," calculation of t_1' and t_2' in the Michelson-Morley experiment, suggest that the same remarks as in the Michelson-Morley experiment hold also for the experiments [25] and [26]. For example, the reflections of light in maser cavities or in Fabry-Perot cavity happen on the moving mirrors as in the Michelson-Morley experiment, which means that the optical paths between the reflecting ends have to be calculated taking into account the Doppler effect, i.e., as in Driscoll's procedure [24]. In fact, the interference of the light waves, e.g., the light waves with close frequencies from two maser cavities in [25], is always determined by their phase difference and not only with their frequencies. Hence, although the measurement of the beat frequency variation is more precise than the measurement of the shift in the number of fringes, it cannot check the validity of the "AT relativity" in the same measure as it can the latter one. Also it has to be noted that the theoretical predictions for the beat frequency variation are strongly dependent on the chosen synchronization. Altogether, contrary to the generally accepted opinion, the experiments [25] and [26] do not confirm the validity of the "AT relativity."

Regarding the "TT relativity," the modern laser versions [25] and [26] of the Michelson-Morley experiment are incomplete experiments (only the beat frequency variation is measured) and cannot be compared with the theory; in the "TT relativity" the same 4D quantity has to be considered in relatively moving IFRs and *the frequency, taken alone, is not a 4D quantity.*

V. THE KENNEDY-THORNDIKE TYPE EXPERIMENTS

In the Kennedy-Thorndike experiment [27] a Michelson interferometer with unequal arm-lengths was employed and they looked for possible diurnal and annual variations in the difference of the optical paths due to the motion of the interferometer with respect to the preferred frame. The measured quantity was, as in the Michelson-Morley experiment, the shift in the number of fringes, and in [27] the authors also found that was no observable fringe shift. We shall not discuss this experiment since the whole consideration is completely the same as in the case of the Michelson-Morley experiment, and, consequently, the same

conclusion holds also here, i.e., the experiment [27] does not agree with the "AT relativity," but directly proves the "TT relativity." A modern version of the Kennedy-Thorndike experiment was carried out in [28], and the authors stated: "We have performed the physically equivalent measurement (with the Kennedy-Thorndike experiment, my remark) by searching for a sidereal 24-h variation in the frequency of a stabilized laser compared with the frequency of a laser locked to a stable cavity." The result was: "No variations were found at the level of 2×10^{-13}." Also they declared: "This represents a 300-fold improvement over the original Kennedy-Thorndike experiment and *allows the Lorentz transformations to be deduced entirely from experiment at an accuracy level of 70 ppm."* (my emphasis) The experiment [28] is of the same type as the experiment [26], and neither the experiment [26] is physically equivalent to the Michelson-Morley experiment, as shown above, nor, contrary to the opinion of the authors of [28], the experiment [28] is physically equivalent to the Kennedy-Thorndike experiment; the measurement of the beat frequency variation is not equivalent to the measurement of the change in the phase difference (in terms of the measurement of the shift in the number of fringes). And, additionally, the Michelson-Morley and the Kennedy-Thorndike experiments can be compared both with the "AT relativity" and the "TT relativity", while the modern laser versions [26], [25] and [28] of these experiments are incomplete experiments from the "TT relativity" viewpoint and cannot be compared with the "TT relativity." Furthermore, the "TT relativity" deals with the covariant 4D Lorentz transformations (Eq.(3) in [*I*] and in Sec.3.3 here) and they cannot be deduced from the experiment [28].

VI. THE IVES-STILLWEL TYPE EXPERIMENTS

Ives and Stilwell [29] performed a precision Doppler effect experiment in which they used a beam of excited hydrogen molecules as a moving light source. The frequencies of the light emitted parallel and antiparallel to the beam direction were measured by a spectograph (at rest in the laboratory). The measured quantity in this experiment is

$$\triangle f/f_0 = (\triangle f_b - \triangle f_r)/f_0, \tag{27}$$

where f_0 is the frequency of the light emitted from resting atoms. $\triangle f_b = |f_b - f_0|$ and $\triangle f_r = |f_r - f_0|$, where f_b is the blue-Doppler-shifted frequency that is emitted in a direction parallel to $\mathbf{v}$ ($\mathbf{v}$ is the velocity of the atoms relative to the laboratory), and f_r is the red-Doppler-shifted frequency that is emitted in a direction opposite to $\mathbf{v}$. The quantity $\triangle f/f_0$ measures the extent to which the frequency of the light from resting atoms fails to lie halfway between the frequencies f_r and f_b. In terms of wavelengths the relation (27) can be written as

$$\triangle \lambda/\lambda_0 = (\triangle \lambda_r - \triangle \lambda_b)/\lambda_0, \tag{28}$$

where $\triangle \lambda_r = |\lambda_r - \lambda_0|$ and $\triangle \lambda_b = |\lambda_b - \lambda_0|$, and, as we said, λ_r and λ_b are the wavelengths shifted due to the Doppler effect to the "red" and "blue" regions of the spectrum. In that way Ives and Stilwell replaced the difficult problem of the precise determination of the wavelength with much simpler problem of the determination of the asymmetry of shifts of the "red" and "blue" shifted lines with respect to the unshifted line. They [29] showed

that the measured results agree with the formula predicted by the traditional formulation of the special relativity, i.e., the "AT relativity," and not with the classical nonrelativistic expression for the Doppler effect. Let us explain it in more detail.

A. The "AT relativity" calculation

In the "AT relativity" one usually starts with the Lorentz transformation of the 4-vector $k^{\mu}(\omega/c, \mathbf{k}=\mathbf{n}\omega/\mathbf{c})$ of the light wave from an IFR S to the relatively moving (along the common $x, x'-$axes) IFR S'. Note that only the "e" coordinatization is used in such traditional treatment. Then the Lorentz transformation in the "e" base of k^{μ} can be written as

$$\omega'/c = \gamma(\omega/c - \beta k^1), k'^1 = \gamma(k^1 - \beta\omega/c), k'^2 = k^2, k'^3 = k^3, \tag{29}$$

or in terms of the unit wave vector $\mathbf{n}$ (which is in the direction of propagation of the wave)

$$\omega' = \gamma\omega(1 - \beta n^1), n'^1 = N(n^1 - \beta), n'^2 = (N/\gamma)n^2, n'^3 = (N/\gamma)n^3, \tag{30}$$

where $N = (1 - \beta n^1)^{-1}$. Now comes the main point in the derivation. Although the Lorentz transformation of the 4-vector k^{μ} from S to S', Eqs.(29) and (30), transforms all four components of k^{μ} the usual "AT relativity" treatment considers the transformation of the temporal part of k^{μ}, i.e., the frequency, as independent of the transformation of the spatial part of k^{μ}, i.e., the unit wave vector $\mathbf{n}$, and thus the "AT relativity" deals with two independent physical phenomena - the Doppler effect and the aberration of light. We note once again that such distinction is possible only in the "e" coordinatization; in the "r" base the metric tensor is not diagonal and consequently the separation of the temporal and spatial parts does not exist. Thus the "AT relativity" calculation is restricted to the "e" base. In agreement with such theoretical treatment the existing experiments (including the modern experiments based on collinear laser spectroscopy; see, e.g., [30–32], or the review [33]) are designed in such a way to measure either the Doppler effect or the aberration of light. Let us write the above transformation in the form from which one can determine the quantities in (28) and then compare with the experiments. The spectograph is at rest in the laboratory (the S frame) and the light source (at rest in the S' frame) is moving with $\mathbf{v}$ relative to S. Then in the usual "AT relativity" approach *only the first relation from (29), or (30), is used,* which means that, in the same way as shown in previous cases, *the "AT relativity" deals with two different quantities in 4D spacetime, here ω and ω'.* Then writting the transformation of the temporal part of k^{μ}, i.e., of ω, in terms of the wavelength λ we find

$$\lambda = \gamma\lambda_0(1 - \beta\cos\theta), \tag{31}$$

where λ is the wavelength received in the laboratory from the moving source (the shifted line), λ_0 $(= \lambda')$ is the natural wavelength (the unshifted line) and θ is the angle of $\mathbf{k}$ relative to the direction of $\mathbf{v}$ as measured in the laboratory. The nonrelativistic treatment of the Doppler effect predicts $\lambda = \lambda_0(1 - \beta\cos\theta)$, and in the classical case the Doppler shift does not exist for $\theta = \pi/2$. This transverse Doppler effect ($\theta = \pi/2$, $\lambda = \gamma\lambda_0$, or $\nu = \nu_0/\gamma$) is always, in the traditional, "AT relativity," approach considered to be a direct consequence

of time dilatation; it is asserted (e.g. [8]) that the frequencies must be related as the inverse of the times in the usual relation for the time dilatation $\triangle t = \triangle t_0 \gamma$. It is usually interpreted [33]: "The Doppler shift experiments ... compare the rates of two "clocks" that are in motion relative to each other. *They measure time dilatation* (my emphasis) and can test the validity of the special relativity in this respect." Similarly it is declared in [30]: "The experiment represents a more than tenfold improvement over other Doppler shift measurements and *verifies the time dilation effect* (my emphasis) at an accuracy level of 2.3ppm." Obviously, as we said, the Doppler shift experiments are theoretically analysed only by means of the "AT relativity," which treats the transformation of the temporal part of k^μ as independent of the transformation of the spatial part of k^μ.

In the Ives and Stilwell type experiments the measurements are conducted at symmetric observation angles θ and $\theta + 180^0$; particularly in [29] θ is chosen to be $\simeq 0^0$. The wavelength in the direction of motion is obtained from (31) as $\lambda_b = \gamma\lambda_0(1-\beta\cos\theta)$, while that one in the opposite direction (the angle $\theta + 180^0$) is $\lambda_r = \gamma\lambda_0(1+\beta\cos\theta)$, and then $\triangle\lambda_b = |\lambda_b - \lambda_0| = |\lambda_0(1-\gamma+\beta\gamma\cos\theta)|$, $\triangle\lambda_r = |\lambda_r - \lambda_0| = |\lambda_0(\gamma - 1 + \beta\gamma\cos\theta)|$, and the difference in shifts is

$$\triangle\,\lambda = \triangle\lambda_r - \triangle\lambda_b = 2\lambda_0(\gamma - 1) \simeq \lambda_0\beta^2, \tag{32}$$

where the last relation holds for $\beta \ll 1$. Note that the redshift due to the transverse Doppler effect ($\lambda_0\beta^2$) is independent on the observation angle θ. In the nonrelativistic case $\triangle\lambda = 0$, the transverse Doppler shift is zero. Ives and Stilwell found the agreement of the experimental results with the relation (32) and not with the classical result $\triangle\lambda = 0$.

However, a more careful analysis shows that the agreement between the "AT relativity" prediction Eq.(32) and the experiments [29] is, contrary to the general belief, only an "apparent" agreement and not the "true" one. This agreement actually happens for the following reasons. First, the theoretical result (32) is obtained in the "e" coordinatization in which one can speak about the frequency ω and the wave vector $\mathbf{k}$ as well-defined quantities. Using the matrix T^μ_ν, which transforms the "e" coordinatization to the "r" coordinatization, $k^\mu_r = T^\mu_\nu k^\nu_e$, (see, [*I*]) one finds $k^0_r = k^0_e - k^1_e - k^2_e - k^3_e$, $\quad k^i_r = k^i_e$, whence we conclude that in the "r" base the theoretical predictions for *the components* of a 4-vector, i.e., for λ, will be quite different than in the "e" base, i.e., than the result (32), and thus not in the agreemement with the experiment [29]. Further, the specific choice of θ ($\theta \simeq 0^0$) in the experiments [29] is the next reason for the agreement with the "AT relativity" result (32). Namely, if $\theta = 0^0$ than $n^1 = 1$, $n^2 = n^3 = 0$, and k^μ is $(\omega/c, \omega/c, 0, 0)$. From (29) or (30) one finds that in S' too $\theta' = 0^0$, $n'^1 = 1$ and $n'^2 = n'^3 = 0$ (the same holds for $\theta = 180^0$, $n^1 = -1$, then $\theta' = 180^0$ and $n'^1 = -1$). In the experiments [29] the emitter is the moving ion (its rest frame is S'), while the observer is the spectrometer at rest in the laboratory (the S frame). Since in [29] the angle of the ray emitted by the ion at rest is chosen to be $\theta' = 0^0$ (180^0), then the angle of this ray measured in the laboratory, where the ion is moving, will be the same $\theta = 0^0$ (180^0). (Similarly happens in the modern versions [30,32] of the Ives-Stilwell experiment; the experiments [30,32] make use of an atomic or ionic beam as a moving light analyzer (the accelerated ion is the "observer") and two collinear laser beams (parallel and antiparallel to the particle beam) as light sources (the emitter), which are at rest in the laboratory.) From this consideration we conclude that in these experiments one can consider only the Doppler effect, that is, the transformation of ω (the temporal part of

k^μ; k^a in the "e" base), and not the aberration of light, i.e., the transformation of $\mathbf{n}$, i.e., $\mathbf{k}$, (the spatial part of k^μ); because of that they found the agreement between the relation (31) (or (32)) with the experiments. However, the relations (29) and (30) reveal that in the case of an arbitrary θ the transformation of the temporal part of k^μ cannot be considered as independent of the transformation of the spatial part, which means that in this case one cannot expect that the relation (32), taken alone, will be in agreement with the experiments performed at some arbitrary θ. Such experiments were, in fact, recently conducted and we discuss them here.

Pobedonostsev and collaborators [34] performed the Ives-Stilwell type experiment but improved the experimental setup and, what is particularly important, the measurements were conducted at symmetric observation angles 77^0 and 257^0. The work was done with a beam of H_2^+ ions at energies $175, 180, 210, 225, 260$ and 275 keV, and the radiation from hydrogen atoms in excited state, which are formed as a result of disintegration of accelerated H_2^+, was observed. The radiation from the moving hydrogen atoms, giving the Doppler shifted lines, was observed together with the radiation from the resting atoms existing in the same working volume, and giving an unshifted line. The similar work was reported in [35] in which a beam of H_3^+ ions at energy 310 keV was used and the measurements were conducted at symmetric observation angles 82^0 and 262^0. The results of the experiments [34] and [35] markedly differed from all previous experiments that were performed at observation angles $\theta = 0^0$ (and 180^0). Therefore in [35] Pobedonostsev declared: *"In comparing the wavelength of Doppler shifted line from a moving emitter with the wavelength of an identical static emitter, the experimental data corroborate the classical formula for the Doppler effect, not the relativistic one."* Thus, instead of to find the "relativistic" result $\triangle\lambda \simeq \lambda_0\beta^2$ (32), (actually the "AT relativity" result), they found the classical result $\triangle\lambda \simeq 0$, i.e., they found that the redshift due to the transverse Doppler effect ($\lambda_0\beta^2$) *is dependent* on the observation angle θ. This experimental result strongly support our assertion that the agreement between the "AT relativity" and the Ives-Stilwell type experiments is only an "apparent" agreement and not the "true" one.

B. The "TT relativity" approach

As already said in the "TT relativity" neither the Doppler effect nor the aberration of light exist separately as well defined physical phenomena. As shown in Sec.3.3 here and in $[I]$, in 4D spacetime the temporal distances (e.g., τ_E and τ_μ) refer to different quantities, which are not connected by the Lorentz transformation; the same happens with ω and ω' as the temporal parts of k^μ. And, as Gamba [6] stated, the fact that the measurements of such quantities were made by *two* observers does not mean that relativity has something to do with the problem. In the "TT relativity" the entire 4D quantity has to be considered both in the theory and in experiments. Therefore, in order to theoretically discuss the experiments of the Ives-Stilwell type we choose as the relevant quantity the wave vector k^a and its square, for which it holds that

$$k^a g_{ab} k^b = 0. \tag{33}$$

First we consider the experiments [34] and [35] since they showed the disagreement with the traditional theory, i.e., with the "AT relativity." The product $k^a g_{ab} k^b$ is a Lorentz scalar

and it is also independent of the choice of the coordinatization. Hence we can calculate that product $k^a g_{ab} k^b$ in the "e" base and in the rest frame of the emitter (the S' frame); the emitter is the ion moving in S, the rest frame of the spectrometer, i.e., in the laboratory frame. Then k^a in the "e" base and in S' is represented by $k'^{\mu} = (\omega'/c)(1, \cos\theta', \sin\theta', 0)$ and $k'^{\mu}k'_{\mu} = 0$. The observer (the spectrometer) in the laboratory frame will look at *the same 4D quantity* k^a and find the Lorentz transformed wave vector k^{μ} as

$$k^{\mu} = [\gamma(\omega'/c)(1 + \beta\cos\theta'), \gamma(\omega'/c)(\cos\theta' + \beta), (\omega'/c)\sin\theta', 0]\,,$$

whence $k^{\mu}k_{\mu}$ is also $= 0$. From that transformation one can find that

$$n^1 = (n'^1 + \beta)/(1 + \beta n'^1), n^2 = n'^2/\gamma(1 + \beta n'^1), n^3 = n'^3/\gamma(1 + \beta n'^1),$$

or that

$$\sin\theta = \sin\theta'/\gamma(1 + \beta\cos\theta'), \cos\theta = (\cos\theta' + \beta)/(1 + \beta\cos\theta'),$$

$$\tan\theta = \sin\theta'/\gamma(\beta + \cos\theta'). \tag{34}$$

The relation (34) reveals that not only ω is changed (the Doppler effect) when going from S' to S than also the angle of $\mathbf{k}$ relative to the direction of $\mathbf{v}$ is changed (the aberration of light). This means that if the observation of the unshifted line (i.e., of the frequency $\omega' = \omega_0$ from the atom at rest) is performed at an observation angle θ' in S', the rest frame of the emitter, then *the same light wave* (from the same but now moving atom) will have the shifted frequency ω and will be seen at an observation angle θ (generally, $\neq \theta'$) in S, the rest frame of the spectrometer. In S' the quantities ω' and θ' define k'^{μ}, and this propagation 4-vector satisfies the relation $k'^{\mu}k'_{\mu} = 0$, which is the representation of the relation (33) in the "e" base and in the S' frame. The quantities ω' and θ' are connected with the corresponding ω and θ, that define the corresponding k^{μ} in S, by the "e" base Lorentz transformation of k'^{μ}. Then k^{μ} is such that it also satisfies the relation $k^{\mu}k_{\mu} = 0$ (the representation of the same relation (33) in the "e" base and now in the S frame). The authors of the experiments [34] (and [35]) made the observation of the radiation from the atom at rest (the unshifted line) and from a moving atom at the same observation angle. The preceding discussion shows that if they succeeded to see ω_0 (λ_0) from the atom at rest at some symmetric observation angles θ' ($\neq 0$) and $\theta' + 180^0$ than they could not see the assymetric Doppler shift (from moving atoms) at the same angles $\theta = \theta'$ (and $\theta' + 180^0$). This was the reason that they detected $\triangle\lambda \simeq 0$ and not $\triangle\lambda \simeq \lambda_0\beta^2$. But we expect that the result $\triangle\lambda \simeq \lambda_0\beta^2$ can be seen if the similar measurements of the frequencies, i.e., the wavelengths, of the radiation from moving atoms would be performed not at $\theta = \theta'$ than at θ determined by the relation (34).

Recently, Bekljamishev [36] came to the same conclusions and explained the results of the experiments [34] and [35] taking into account the aberration of light together with the Doppler effect. It is argued in [36] that Eq.(31) for the Doppler effect can be realized only when the condition for the aberration angle is fulfilled,

$$\triangle\,\theta = \beta\sin\theta', \tag{35}$$

where $\triangle\theta = \theta' - \theta$, and β is taken to be $\beta \ll 1$. The relation (35) directly follows from the expression for $\sin\theta$ in (34) taking that $\beta \ll 1$. The assymetric shift will be seen when the

collimator assembly is tilted at a velocity dependent angle $\triangle\theta$. Instead of to work, as usual, with the arms of the collimator at fixed angles θ and $\theta + 180^0$, Bekljamishev [36] proposed that the collimator assembly must be constructed in such a way that there is the possibility of the correction of the observation angles independently for both arms; for example, the arm at angle θ ($\theta + 180^0$) has to be tilted clockwise (counter-clockwise) by the aberration angle $\triangle\theta$. Otherwise the assymetry in the Doppler shifts will not be observed. Thus the experiments [34] and [35] would need to be repeated taking into account Bekljamishev's proposition. The positive result for the Doppler shift $\triangle\lambda$ (32), when the condition for the aberration angle $\triangle\theta$ (35) is fulfilled, will definitely show that it is not possible to treat the Doppler effect and the aberration of light as separate, well-defined, effects, i.e., that it is the "TT relativity," and not the "AT relativity," which correctly explains the experiments that test the special relativity.

VII. CONCLUSIONS AND DISCUSSION

The analysis of the experiments which test the special relativity shows that they agree with the predictions of the "TT relativity" and not, as usually supposed, with those of the "AT relativity."

In the "muon" experiment the fluxes of muons on a mountain, N_m, and at sea level, N_s, are measured. The "AT relativity" predicts different values of the flux N_s (for the same measured N_m) in different synchronizations, but the measured N_s is independent of the chosen coordinatization. Further, for some synchronizations these predicted values of the flux at sea level N_s are quite different than the measured ones. The reason for such disagreement, as explained in the text, is that in the usual, "AT relativity," analysis of the "muon" experiment, for example, the lifetimes τ_E and τ_μ are considered to refer to the same temporal distance (the same quantity) measured by the observers in two relatively moving IFRs. But the transformation connecting τ_E and τ_μ (the dilatation of time) is only *a part* of the Lorentz transformation written in the "e" base, and, actually, τ_E and τ_μ refer to different quantities in 4D spacetime. Although their measurements were made by *two* observers, the relativity has nothing to do with the problem, since τ_E and τ_μ are different 4D quantities. *The "TT relativity," in contrast to the "AT relativity," completely agrees with the "muon" experiments in all IFRs and all possible coordinatizations.* In the "TT relativity" *the same 4D quantity* is considered in different IFRs and different coordinatizations; instead of to work with τ_E and τ_μ the "TT relativity" deals with the spacetime length l and formulate the radioactive-decay law in terms of covariantly defined quantities.

In the Michelson-Morley experiment the traditional, "AT relativity," derivation of the fring shift $\triangle N$ deals only with the calculation, in the "e" base, of path lengths (optical or geometrical) in S and S', or, in other words, with the calculation of t_1 and t_2 (in S and S'), which are the times required for the complete trips OM_1O and OM_2O along the arms of the Michelson-Morley interferometer. The null fringe shift obtained with such calculation is only in an "apparent," not "true," agreement with the observed null fringe shift, since this agreement was obtained by an incorrect procedure. Namely it is supposed in such derivation that, e.g., t_1 and t'_1 refer to the same quantity measured by the observers in relatively moving IFRs S and S' that are connected by the Lorentz transformation. However the relation $t'_1 = \gamma t_1$ is not the Lorentz transformation of some 4D quantity, and t'_1 and t_1 do

not correspond to the same 4D quantity considered in S' and S respectively. The improved "AT relativity" calculation from [24] (again in the "e" base) determines the increment of phase along some path not only by the segment of geometric path length than also by the wavelength in that segment, and finds a non-null fringe shift. But we show that the non-null theoretical result for the fringe shift, which is obtained in [24], is a consequence of the fact that again two different quantities $k_e^0 l_{0e}$ and $k_e'^0 l_{0e}'$ (only the parts of the covariantly defined phase (22) $\phi = k^a g_{ab} l^b$) are considered to refer to the same 4D quantity, and thus that these two quantities are connected by the Lorentz transformation. The "TT relativity," in contrast to the "AT relativity" calculations, deals always with the covariantly defined 4D quantities (in the Michelson-Morley experiment, e.g., the covariantly defined phase (22) $\phi = k^a g_{ab} l^b$), which *are* connected by the Lorentz transformation. *The "TT relativity" calculations yields the observed null fringe shift and that result holds for all IFRs and all coordinatizations.*

The same conclusions can be drawn for the Kennedy-Thorndike type experiments.

In the Ives-Stilwell type experiments the agreement between the "AT relativity" calculation for the Doppler effect and the experiments is again only an "apparent" agreement and not the "true" one. Namely the transverse Doppler shift ($\lambda_0 \beta^2$, (32)) is obtained in the "e" coordinatization in which one can speak about the frequency ω and the wave vector $\mathbf{k}$ as well-defined quantities. Further in the usual "AT relativity" approach only the transformation of ω (the temporal part of k^μ) is considered, while the aberration of light, i.e., the transformation of $\mathbf{n}$, i.e., $\mathbf{k}$, (the spatial part of k^μ) is neglected. Thus in this case too the "AT relativity" deals with two different quantities in 4D spacetime, ω and ω', which are not connected by the Lorentz transformation. However, for the specific choice of the observation angles $\theta' = 0^0$ (180^0) in S' (the rest frame of the emitter), one finds from the transformation of k^μ that θ in S is again $= 0^0$ (180^0). Since in the experiments [29], and its modern versions [30,32], just such angles were chosen, it was possible to consider only the transformation of ω, i.e., only the Doppler effect, and not the concomitant aberration of light, and because of that they found the agreement between the relation (31) (or (32)) and the experiments. When the experiments were performed at observation angles $\theta \neq 0^0$ (and 180^0), as in [34] and [35], the results disagreed with the "AT relativity" calculation which takes into account only the transformation of ω, i.e., only the Doppler effect. The "TT relativity" calculation considers *the same 4D quantity* the wave vector k^a (or its square) in S and S', i.e., it considers the Doppler effect and the aberration of light together as unseparated phenomena. The results of such calculation agrees with the experiments [29] and [30,32] (made at $\theta = 0^0$ (180^0)), but also predict the positive result for the Doppler shift $\triangle\lambda$ (32) in the experiments of the type [34] and [35], if the condition for the aberration angle $\triangle\theta$ (35) is fulfilled, which agrees with Bekljamishev's explanation [36] of the experiments [34] and [35].

The discussion in this paper clearly shows that *the "TT relativity" completely agrees with all considered experiments, in all IFRs and all possible coordinatizations,* while the "AT relativity" appears as an unsatisfactory relativistic theory. These results are directly contrary to the generally accepted opinion about the validity of the "AT relativity."

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